

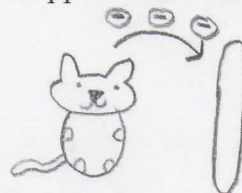
Mock exam I

Print off your own copy of the mock exam and do your best to complete it before the mock exam session on Wednesday, January 29, at 6:30 PM in EB128. We will not take any time during the mock exam session to complete the mock exam.

This mock exam will be most beneficial to you if you complete it under testing conditions. It should take you about 1 hour and 15 minutes to complete, but keep in mind that your class exam will be only 50 minutes. Use the equation sheet provided by Dr. Wenger on Blackboard and the same calculator that you plan to use during the actual exam. If you get stuck, try to use the strategy in the document called "How to solve a physics problem" to help you. Do not use your textbook or notes from class until you have attempted the problems in the mock exam on your own at least once.

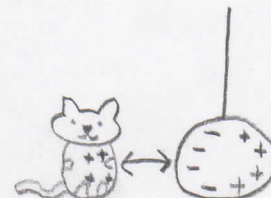
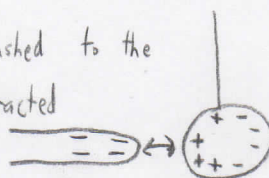
1. An electrically neutral conducting metal ball hangs from a surface by an insulating string such that the ball is free to swing in any direction. Then, a rubber rod is rubbed against the fur of a housecat so that electrons are transferred from the cat to the rod. The rod is brought near the hanging ball but does not make contact with it. After the rod is removed, the cat is brought near the hanging ball but does not make contact with it. What happens to the ball in each case?

- A. The ball is repulsed by the rod and is attracted by the cat.
- B. The ball is attracted by both the cat and the rod.
- C. The ball is repulsed by both the cat and the rod.
- D. The ball is attracted by the rod but does not respond to the cat.
- E. The ball is repeatedly swatted by the cat and falls off of the string, ruining the experiment. Cats can't be expected to follow rigorous experimental protocols.



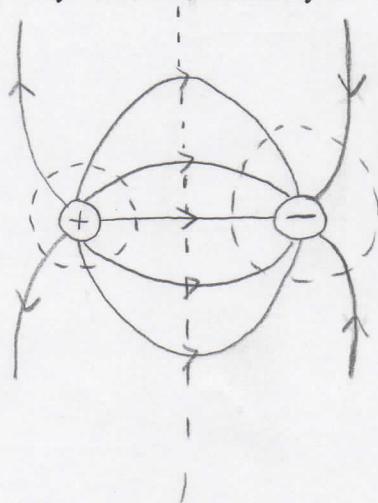
Charging by induction - like charges are pushed to the opposite, and closer opposite charges are attracted

2. Which of the following statements are true?



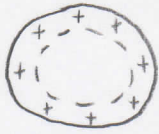
- I. The density of electric field lines in a region of space is proportional to the strength of the electric field in that region of space.
- II. Electric field lines always point from negative charges to positive charges.
- III. Electric field lines are always oriented normally to equipotential surfaces.

- A. I only
- B. III only
- C. I and II only
- D. I and III only
- E. I, II, and III

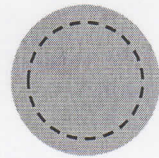


- I. True. More lines means stronger field.
- II. False. Electric field lines show what happens to a positive test charge, so they point positive \rightarrow negative.
- III. True. Dashed lines are examples of equipotential surfaces.

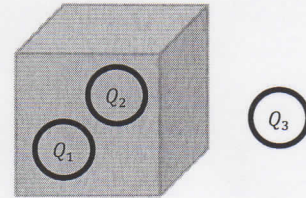
3. F (T/F) In the spherical conductor with charge $+Q$ shown to the right, Gauss's law predicts that the electric flux through the dashed surface is proportional to Q .



Like charges repel, so all charge in a conductor is spread evenly over the surface. There is no charge inside the dashed line, so $\Phi_E = \frac{q}{\epsilon_0} = \frac{0}{\epsilon_0} = 0$.



4. Consider a cubical Gaussian surface with length 3 cm , as shown to the right. Particles Q_1 and Q_2 (with charges $+9.2 \text{ nC}$ and -4.3 nC , respectively) are contained within the Gaussian surface, and particle Q_3 (with charge $+6.1 \text{ nC}$) lies outside of it. Calculate the sum of the normal components of the electric field through the Gaussian surface.



$$\Phi_E = \underbrace{\sum E \cos \phi}_{\text{Question asks for this}} \underbrace{\Delta A}_{\text{Surface area of cube}} = \frac{q}{\epsilon_0}$$

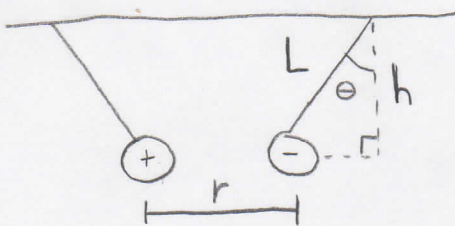
Gauss's law

We only care about charges inside the Gaussian surface.

$$\sum E \cos \phi = \frac{q}{\epsilon_0 \sum \Delta A} = \frac{Q_1 + Q_2}{\epsilon_0 \cdot 6d^2} = \frac{(9.2 - 4.3) \text{ nC}}{(8.854 \cdot 10^{-12} \text{ C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2}) \cdot 6(3 \text{ cm})^2} = 102 \text{ N} \cdot \text{C}^{-1}$$

6 faces, area of each face is d^2

5. Two spherical conductors, each with a mass of 3.0 kg , hang from a surface by an insulating string of length 1.0 cm . Then, one conductor is given a charge of $+2.0 \text{ } \mu\text{C}$, and the other is given a charge of $-2.0 \text{ } \mu\text{C}$. At equilibrium, the distance between the conductors is 46 mm . Calculate the distance between the conductors and the surface above them at equilibrium. (Hint: while this problem can be solved using energy, it is mathematically easier in this case to use Newton's laws at equilibrium to find the angular displacement of the string and then use trigonometry to find the desired distance.)



Coulomb Force $F_E = \frac{k|q_1q_2|}{r^2}$ since $|q_1| = |q_2|$

Use Newton's laws in x and y direction.

$$\begin{aligned} \sum F_x &= F_T \sin \theta - F_E = 0 \\ \sum F_y &= F_T \cos \theta - F_G = 0 \end{aligned}$$

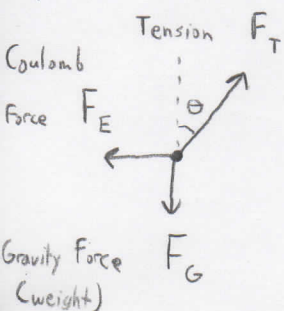
divide \rightarrow

$$\tan \theta = \frac{F_E}{F_G} = \frac{k|q|^2}{mgr^2}$$

$$\theta = \tan^{-1} \left(\frac{8.988 \cdot 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \cdot (2.0 \text{ } \mu\text{C})^2}{30 \text{ kg} \cdot 9.807 \text{ m} \cdot \text{s}^{-2} \cdot (46 \text{ mm})^2} \right) = 30.01^\circ$$

$$h = L \cos \theta = (10 \text{ mm}) \cos (30.01^\circ) = 8.7 \text{ mm}$$

Free-body diagram for negative charge (mirror-image for positive charge)



6. T (T/F) The product of one farad and one volt is one coulomb.

$1 \text{ F} \cdot 1 \text{ V} = 1 \text{ C}$ or use equation for charge stored on a capacitor.

$1 \frac{\text{C}}{\text{V}} \cdot 1 \text{ V} = 1 \text{ C}$ $q = CV$ (Coulomb) = (farad)(volt)

7. A uniform electric field exists between two charged metal plates. An electron is released from rest between the plates and accelerates toward one plate, achieving a final speed v_0 . Then, the space in between the plates is filled with chlorine gas, which has a dielectric constant κ of 2.0. A second electron is released from rest in the same location as the first. What is the final speed of the second electron compared to v_0 ? Assume that the gas does not interact with the electron during its movement.

Electric potential energy is converted to kinetic energy

- A. The final speed is larger than v_0 by a factor of 2
- B. The final speed is larger than v_0 by a factor of $\sqrt{2}$
- C. The final speed is equal to v_0
- D. The final speed is smaller than v_0 by a factor of $\sqrt{2}$
- E. The final speed is smaller than v_0 by a factor of 2

$U_E = q\Delta V \Rightarrow K = \frac{1}{2}mv^2$

Second electron:

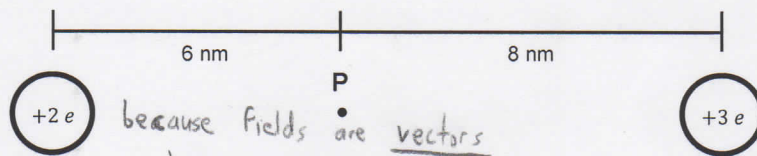
First electron:

$v_1 = v_0 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2q\Delta V}{m}}$

Since the battery is disconnected, the charge on the capacitors remains constant, and ΔV decreases by a factor of κ

$v_2 = \sqrt{\frac{1}{2} \frac{2q\Delta V}{m}} = \frac{v_0}{\sqrt{2}}$

8. Two charged particles are arranged on a straight line as shown below. The particles have charges of $+2e$ and $+3e$, where e is fundamental charge. Calculate a) the electric field at point P and b) the electric potential at point P. If a third particle with a charge of $-2e$ were to be placed at point P, calculate c) the net electrostatic force on the particle and d) the electric potential energy of the particle.



a) $\sum \vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{k|q_1|}{r_1^2} - \frac{k|q_2|}{r_2^2}$
 $\sum \vec{E} = (8.988 \cdot 10^9 \text{ N}\cdot\text{m}^2\cdot\text{C}^{-2}) \left(\frac{2 \cdot 1.602 \cdot 10^{-19} \text{ C}}{(6 \text{ nm})^2} - \frac{3 \cdot 1.602 \cdot 10^{-19} \text{ C}}{(8 \text{ nm})^2} \right)$

c) $\sum \vec{F} = q_0 \vec{E}$
 $\sum \vec{F} = -2 \cdot 1.602 \cdot 10^{-19} \text{ C} \times 1.250 \cdot 10^7 \text{ N}\cdot\text{C}^{-1}$

$\sum \vec{E} = +1.2 \cdot 10^7 \text{ N}\cdot\text{C}^{-1}$ because potentials are scalars

$\sum \vec{F} = -4.0 \cdot 10^{-12} \text{ N}$
 Why negative? Test charge is negative.

b) $\sum V = V_1 + V_2 = \frac{k|q_1|}{r_1} + \frac{k|q_2|}{r_2}$
 $\sum V = (8.988 \cdot 10^9 \text{ N}\cdot\text{m}^2\cdot\text{C}^{-2}) \left(\frac{2 \cdot 1.602 \cdot 10^{-19} \text{ C}}{6 \text{ nm}} + \frac{3 \cdot 1.602 \cdot 10^{-19} \text{ C}}{8 \text{ nm}} \right)$

d) $U_E = q_0 V$
 $U_E = -2 \cdot 1.602 \cdot 10^{-19} \text{ C} \times 1.020 \text{ V}$

$\sum V = 1.0 \text{ V}$

$U_E = -3.3 \cdot 10^{-19} \text{ J}$

9. The plasma membrane of a neuronal axon acts like a parallel plate capacitor with a dielectric constant of 5.0. Consider a 25-square millimeter section of an axon membrane with a thickness of 10 nm. If the total charge outside of that section of the membrane is 7.8 nC , calculate the potential difference across the membrane ~~and the energy density within the membrane.~~

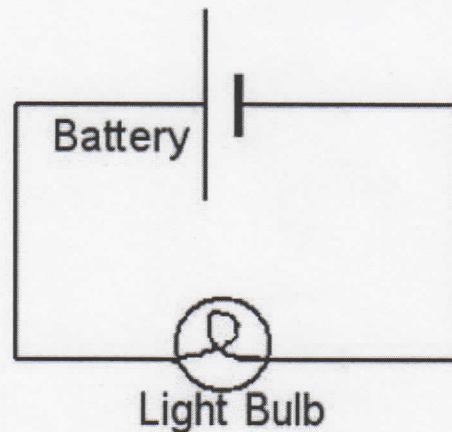
$$a) V = \frac{q}{C} = \frac{q d}{\kappa \epsilon_0 A} = \frac{7.8 \text{ nC} \cdot 10 \text{ nm}}{5.0 \cdot 8.854 \cdot 10^{-12} \text{ C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2} \cdot (25 \text{ mm}^2)} = 0.070 \text{ V} \equiv 70 \text{ mV}$$

$$b) \frac{\text{Energy from } q \text{ C}}{\text{Volume } A \cdot d} = \frac{\kappa \epsilon_0 A}{d} \quad \text{and} \quad q = CV$$

10. F (T/F) Ohm's law states that the potential difference across an element in a circuit is equal to the product of the electric current through the element and the resistance of the element. *For all materials.*

False. Ohm's law only applies to certain materials. Superconductors are examples of non ohmic devices.

11. In the simple circuit shown to the right, what is the direction of the conventional current and the direction of the motion of the charge carriers?



- A. The conventional current is directed clockwise, and the charge carriers are moving counterclockwise.
- B. The conventional current is directed clockwise, and the charge carriers are moving clockwise.
- C. The conventional current is directed counterclockwise, and the charge carriers are moving clockwise.
- D. The conventional current is directed counterclockwise, and the charge carriers are moving counterclockwise.
- E. There is no current in this circuit.

Battery symbol means $+ | \quad | -$

Conventional current is defined as the direction of motion of positive charge.

Actual charge carriers are electrons with negative charge, so they flow in the opposite direction.

12. A resistor shaped like a cylinder with radius 6.1 μm and length 2.0 cm is used in an alternating current circuit with a peak current of 1.5 A. The average power consumed by the circuit is 10.8 W. A table of resistivities of various metals is given in the table to the right. From what material is the resistor made?

Metal	Resistivity ($10^{-8} \Omega \cdot \text{m}$)
Copper	1.7
Aluminum	2.8
Tungsten	5.6
Iron	9.7
Black Sabbath	13

AC circuit means average power is given by $\bar{P} = I_{\text{rms}} V_{\text{rms}} = I_{\text{rms}}^2 R = \left(\frac{I_p}{\sqrt{2}}\right)^2 R$

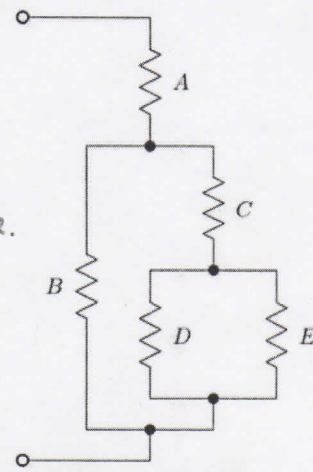
Resistivity is defined by $R = \rho \frac{L}{A}$. So,

$$\bar{P} = \left(\frac{I_p}{\sqrt{2}}\right)^2 R = \left(\frac{I_p}{\sqrt{2}}\right)^2 \rho \frac{L}{A}$$

$$\rho = \frac{\sqrt{2}^2 \bar{P} A}{I_p^2 L} = \frac{2 \bar{P} \pi r^2}{I_p^2 L} = \frac{2 \pi (10.8 \text{ W})(6.1 \mu\text{m})^2}{(1.5 \text{ A})^2 (2.0 \text{ cm})} = 5.6 \cdot 10^{-8} \Omega \cdot \text{m}$$

The resistor is made from tungsten.

13. Find an equivalent resistor to the system of resistors shown at the right. The resistors shown to the right all have a resistance of 12 Ω . Find a) an equivalent resistor for the system of resistors and b) current through each resistor if a 9 V battery were hooked up to the system of resistors.



a) Use rules for resistors in series and in parallel one-at-a-time.

parallel $R_{DE} = \left(\frac{1}{R_D} + \frac{1}{R_E}\right)^{-1} = \frac{R_D R_E}{R_D + R_E} = \frac{(12 \Omega)^2}{2 \cdot 12 \Omega} = 6 \Omega$

series $R_{CDE} = R_C + R_{DE} = 12 \Omega + 6 \Omega = 18 \Omega$

parallel $R_{BCDE} = \left(\frac{1}{R_B} + \frac{1}{R_{CDE}}\right)^{-1} = \frac{R_B R_{CDE}}{R_B + R_{CDE}} = \frac{(12 \Omega)(18 \Omega)}{12 \Omega + 18 \Omega} = 7.2 \Omega$

series $R_{ABCDE} = R_A + R_{BCDE} = 12 \Omega + 7.2 \Omega = 19.2 \Omega$

b) Remember resistors in series have same current and resistors in parallel have same voltage.

$$I_A = \frac{V_{ABCDE}}{R_{ABCDE}} = \frac{9 \text{ V}}{19.2 \Omega} = 0.47 \text{ A}$$

$$I_B = \frac{V_{BCDE}}{R_B} = \frac{V_{ABCDE} - V_A}{R_B} = \frac{V_{ABCDE} - I_A R_A}{R_B} = \frac{9 \text{ V} - 0.47 \text{ A} \cdot 12 \Omega}{12 \Omega}$$

$$I_B = 0.28 \text{ A}$$

$$I_C = \frac{V_{CDE}}{R_{CDE}} = \frac{9 \text{ V} - 0.47 \text{ A} \cdot 12 \Omega}{18 \Omega}$$

$$I_C = \frac{V_{BCDE}}{R_{CDE}} = \frac{9V - 0.47A \cdot 12\Omega}{18\Omega} = 0.19A$$

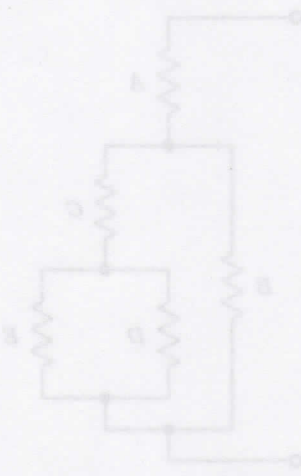
$$I_D = I_E = \frac{V_{DE}}{R_D} = \frac{V_{BCDE} - V_C}{R_D} = \frac{V_{BCDE} - I_C R_C}{R_D}$$

$$I_D = I_E = \frac{9V - 0.47A \cdot 12\Omega - 0.19A \cdot 12\Omega}{12\Omega} = 0.09A$$

Metal	Resistivity (10 ⁻⁸ Ω·m)
Copper	1.7
Aluminum	2.8
Iron	10
Black Sapphire	13

12. A resistor shaped like a cylinder with radius 0.1 mm and length 2.0 cm is used in an alternating current circuit with a peak current of 1.5 A. W. A table of resistivities is given in the table to the right. From what material is the resistor made?

13. Find an equivalent resistor to the system of resistors shown at the right. The resistors shown to the right all have a resistance of 12 Ω. Find a) an equivalent resistor for the system of resistors and b) current through each resistor if a 9 V battery were hooked up to the system of resistors.



a) The resistors in parallel are R_B and R_C .
 $R_{BC} = \left(\frac{1}{R_B} + \frac{1}{R_C} \right)^{-1} = \left(\frac{1}{12\Omega} + \frac{1}{12\Omega} \right)^{-1} = 6\Omega$
 $R_{CD} = R_C + R_D = 12\Omega + 12\Omega = 24\Omega$
 $R_{DE} = \left(\frac{1}{R_D} + \frac{1}{R_E} \right)^{-1} = \left(\frac{1}{12\Omega} + \frac{1}{12\Omega} \right)^{-1} = 6\Omega$
 $R_{FG} = R_F + R_G = 12\Omega + 12\Omega = 24\Omega$
 $R_{HI} = \left(\frac{1}{R_H} + \frac{1}{R_I} \right)^{-1} = \left(\frac{1}{12\Omega} + \frac{1}{12\Omega} \right)^{-1} = 6\Omega$
 $R_{JK} = R_J + R_K = 12\Omega + 12\Omega = 24\Omega$
 $R_{LM} = \left(\frac{1}{R_L} + \frac{1}{R_M} \right)^{-1} = \left(\frac{1}{12\Omega} + \frac{1}{12\Omega} \right)^{-1} = 6\Omega$
 $R_{NO} = R_N + R_O = 12\Omega + 12\Omega = 24\Omega$
 $R_{PQ} = \left(\frac{1}{R_P} + \frac{1}{R_Q} \right)^{-1} = \left(\frac{1}{12\Omega} + \frac{1}{12\Omega} \right)^{-1} = 6\Omega$
 $R_{RS} = R_R + R_S = 12\Omega + 12\Omega = 24\Omega$
 $R_{TU} = \left(\frac{1}{R_T} + \frac{1}{R_U} \right)^{-1} = \left(\frac{1}{12\Omega} + \frac{1}{12\Omega} \right)^{-1} = 6\Omega$
 $R_{VW} = R_V + R_W = 12\Omega + 12\Omega = 24\Omega$
 $R_{XY} = \left(\frac{1}{R_X} + \frac{1}{R_Y} \right)^{-1} = \left(\frac{1}{12\Omega} + \frac{1}{12\Omega} \right)^{-1} = 6\Omega$
 $R_{Z1} = R_Z + R_{12} = 12\Omega + 12\Omega = 24\Omega$

b) Resistor resistors in series have same current and resistors in parallel have same voltage.
 $I_A = \frac{V_{source}}{R_{total}} = \frac{9V}{11.5\Omega} = 0.77A$
 $I_B = \frac{V_{BC}}{R_B} = \frac{9V - V_{R_A}}{12\Omega} = \frac{9V - 0.77A \cdot 12\Omega}{12\Omega} = 0.58A$