PH202-1G
Spring 2014

Worksheet April 9

Rutherford

Describe Rutherford's gold foil experiment. What did it reveal about the nature of the atom?

Rutherford fired alpha particles at thin gold sheets and observed that the alpha particles were scattered. This discredited the plum pludding model and revealed the presence of the nucleus.

Spectral lines

Describe spectral lines. Why do they form?

Spectral lines form when gaseous atoms emit light due to energy transitions

Give a mathematical expression for the wavelength of a spectral line of hydrogen.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \quad \text{where} \quad R_H = 1.09678 \cdot 10^7 \ m^{-1} \quad \text{and} \quad n > m$$

What three spectral line series are important for the hydrogen atom? Which one involves spectral lines in the visual range?

Lyman series $(m = 1)$, Balmer series $(m = 2)$, and Paschen series $(m = 3)$

Lines in the Balmer series are in the visible range.

Bohr model

Describe the Bohr model of the atom qualitatively.

Electrons orbit the nucleus in uniform circular motion at quantized radii.

What is the force acting on the electron in the Bohr model of the atom?

Centripetal force and electrostatic force $\quad m\frac{v^2}{r} = k_c \frac{Ze^2}{r^2}$

What is the total energy of the electron in the Bohr model?

$$E = KE + EPE = \frac{1}{2}mv^2 - k_c \frac{Ze^2}{r} = -k_c \frac{Ze^2}{2r}$$

What is the angular momentum of the electron in the Bohr model? Why is it significant?

$$L = mvr = n\frac{h}{2\pi}$$

It is significant because it is quantized in the Bohr model.

Give a mathematical expression for the radius of an electron orbit in the Bohr model.

$$r = \frac{h^2}{4\pi^2 m k_c e^2} \frac{n^2}{Z}$$

Give a mathematical expression for the energy levels of an electron in the Bohr model.

$$E = -\frac{2\pi^2 m k_c^2 e^4}{h^2} \frac{Z^2}{n^2}$$

n is a positive integer. It cannot be zero.

Describe the spectral lines of hydrogen according to the Bohr model.

$$E = hf = \frac{hc}{\lambda} = -13.6 \text{ eV} \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$$

for the transition from n_1 to n_2

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What is ionization energy? Give a mathematical expression for it in the Bohr model.

Energy required to remove an electron from a gaseous atom. Transition from $n_1 = 1$ to $n_2 = \infty$

What contribution did de Broglie make to the Bohr model?

$\frac{1}{\infty} = 0$, so $E = +\frac{13.6\text{ eV}}{1^2} = 13.6\text{ eV}$

de Broglie's wave hypothesis applied to the electron in an atom justifies the assumption that angular momentum is quantized.

Problems

1. One spectral line of hydrogen in the Paschen ($m = 3$) series has a frequency of 234 THz. In which excited state was the electron whose transition produces this spectral line?

$1\text{ THz} = 10^{12}\text{ Hz}$

$c = \lambda f$ $\qquad \frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$

$$\frac{1}{n^2} = \frac{1}{m^2} - \frac{1}{\lambda R_H} = \frac{1}{m^2} - \frac{f}{c R_H} = \frac{1}{3^2} - \frac{234\text{ THz}}{3.00\cdot10^8\text{ m}\cdot\text{s}^{-1}\,(1.10\cdot10^7\text{ m}^{-1})}$$

$$\frac{1}{n^2} = 0.040 \Rightarrow n = \sqrt{25} = 5$$

The ground state is $n = 1$, so this represents the fourth excited state.

2. Use the Bohr model to calculate the angular momentum, orbital radius, and energy of an electron in the second excited state.

Second excited state: $n = 3$ $\qquad$ For hydrogen, $Z = 1$

$$L = n\frac{h}{2\pi} = \frac{3}{2\pi}\cdot 6.63\cdot10^{-34}\text{ J}\cdot\text{s} = 3.16\cdot10^{-34}\text{ J}\cdot\text{s or } 3.16\cdot10^{-34}\text{ N}\cdot\text{m}\cdot\text{s}$$

$$r = \frac{h^2}{4\pi^2 m k_c e^2}\frac{n^2}{Z}$$

$$r = \frac{(6.63\cdot10^{-34}\text{ J}\cdot\text{s})^2}{4\pi^2(9.11\cdot10^{-31}\text{ kg})(8.99\cdot10^9\text{ N}\cdot\text{m}^2\cdot\text{C}^{-2})(1.60\cdot10^{-19}\text{ C})^2}\frac{3^2}{1} = 4.76\cdot10^{-10}\text{ m or } 4.76\text{ Å}$$

$$E = -\frac{2\pi^2 m k_c^2 e^4}{h^2}\frac{Z^2}{n^2}$$

$$E = \frac{2\pi^2(9.11\cdot10^{-31}\text{ kg})(8.99\cdot10^9\text{ N}\cdot\text{m}^2\cdot\text{C}^{-2})^2(1.60\cdot10^{-19}\text{ C})^4}{(6.63\cdot10^{-34}\text{ J}\cdot\text{s})^2}\frac{1^2}{3^2} = -2.42\cdot10^{-19}\text{ J or } -1.51\text{ eV}$$