PH202-1G
Spring 2014

Worksheet April 14

Quantum mechanical model of hydrogen

List the four quantum numbers associated with the hydrogen atom and the range of values each can take.

Principal quantum number n — any integer greater than zero
Orbital quantum number ℓ — any integer less than n, including zero
Magnetic quantum number m_ℓ — any integer between $-\ell$ and $+\ell$
Spin quantum number m_s — $+\frac{1}{2}$ or $-\frac{1}{2}$

Give a mathematical expression for the total angular momentum and the *z* component of the angular momentum in the hydrogen atom.

$$L = \sqrt{\ell(\ell+1)}\,\frac{h}{2\pi} \qquad L_z = m_\ell \frac{h}{2\pi}$$

Describe the two main differences between the quantum mechanical model and the Bohr model of the hydrogen atom. What values can the angular momentum take? How does the model describe electrons?

Bohr – L has one value per energy level and cannot be zero; electrons have defined orbits. Quantum mechanics – L can have more than one value per energy level and can be zero; electrons have a cloud of probability where they are most likely to be

Describe the Pauli exclusion principle.

No two electrons in the same atom can have the same set of quantum numbers

What is the ground state electron configuration of an atom?

Number of electrons in each subshell when all electrons are in the lowest possible energy level.

X-rays

Describe how electron bombardment can be used to produce x-rays.

Incoming electrons transfer energy to $n = 1$ electrons and knock them off. Electrons in higher energy levels transition to $n = 1$, producing a photon.

Give a mathematical expression for the wavelength of an x-ray produced by electron bombardment.

$$\frac{1}{\lambda} = (Z-1)^2 R_H \left(1 - \frac{1}{n^2}\right)$$

Give a mathematical expression for the cutoff wavelength in the x-ray spectrum of a material.

$$\lambda_0 = \frac{hc}{eV}$$

Lasers

Describe the difference between spontaneous emission and stimulated emission.

Spontaneous emission will occur naturally over time and produces one photon. Stimulated emission requires an incoming photon with the same energy as the gap between the energy levels and produces two coherent photons.

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What is population inversion? What is a metastable state? Why are they important in the operation of a laser?

Population inversion – more atoms in excited state than in ground state. Metastable state – excited state that is unlikely to decay to the ground state. Both make it more likely that stimulated emission will occur over spontaneous emission so that emitted light is coherent.

Describe the operation of a helium-neon laser.

Helium is excited by an external energy source and collides with neon atoms, transfering energy so that neon is in a metastable state. Neon can only decay by stimulated emission, so the resulting light is coherent.

Problems

1. A hydrogen atom has been excited to the $n = 4$ state. What are the maximum values of the total angular momentum and the z component of the angular momentum for this atom?

$n = 4$, so the maximum values of ℓ and m_ℓ are 3 and 3.

$$L = \sqrt{\ell(\ell+1)}\frac{h}{2\pi} = \sqrt{3(3+1)}\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi} = 3.65\times10^{-34}\ \text{J}\cdot\text{s} \text{ or } 2.28\times10^{-15}\ \text{eV}\cdot\text{s}$$

$$L_z = m_\ell \frac{h}{2\pi} = 3\ \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi} = 3.16\times10^{-34}\ \text{J}\cdot\text{s} \text{ or } 1.97\times10^{-15}\ \text{eV}\cdot\text{s}$$

2. The electron configuration of a cobalt atom is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^6\ 4p^1$. Is this cobalt atom in a ground state or an excited state?

The ground state of cobalt is $1_s^2\ 2_s^2\ 2_p^6\ 3_s^2\ 3_p^6\ 4_s^2\ 3_d^7$. Since a 4_p electron has more energy than a 3_d electron $(4>3)$, this is an excited state.

3. An electron is accelerated through a potential difference of 47.0 kV and collides with a sample of manganese (atomic number 25). What is the cutoff wavelength for this setup? What is the wavelength of the x-ray produced when the transitioning electron was originally in the $n = 3$ shell?

$$\lambda_0 = \frac{hc}{eV} = \frac{(4.14\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m}\cdot\text{s}^{-1})}{47.0\ \text{keV}} = 2.64\times10^{-11}\ \text{m}$$

Remember, an electron volt is the energy of an electron accelerated through one volt. So, accelerating through 47 kV is 47 keV.

$$\frac{1}{\lambda} = R_H (Z-1)^2\left(1-\frac{1}{n^2}\right)$$

$$\lambda = \left[1.097\times10^{7}\ \text{m}^{-1}(25-1)^2\left(1-\frac{1}{3^2}\right)\right]^{-1} = 1.78\times10^{-10}\ \text{m}$$