PH202-1G
Spring 2014

Worksheet February 3

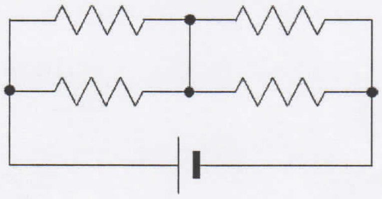
Resistors wired in series and parallel

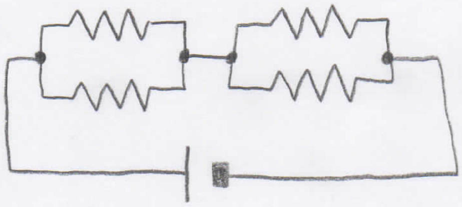
Consider resistors wired in series and in parallel. In each case, is the current through each resistor, the voltage across each resistor, and the power consumed by each resistor the same or additive?

In series, current is the same, and voltage and power are additive.

In parallel, voltage is the same, and current and power are additive.

Redraw the circuit below to emphasize the series and parallel wiring between the resistors.





Internal resistance

What is internal resistance? How does it affect the voltage provided by a device such as a battery?

Internal resistance is the resistance of the internal components of a device which provides an emf. It reduces the voltage provided by such a device.

Does the terminal voltage of a battery depend on a quantity other than internal resistance?

Yes. Terminal voltage depends on the current through the circuit. $V = \varepsilon - Ir$

Kirchhoff's rule

Describe the junction rule in your own words. Then, give a mathematical expression for the rule.

Current going into a junction must equal current coming out of it. $\sum I_{in} = \sum I_{out}$

Describe the loop rule in your own words. Then, give a mathematical expression for the rule.

Traveling around any loop in a circuit will not increase or decrease the potential. $\sum V_{loop} = 0$

How do you decide which direction the current flows in a branch of a circuit?

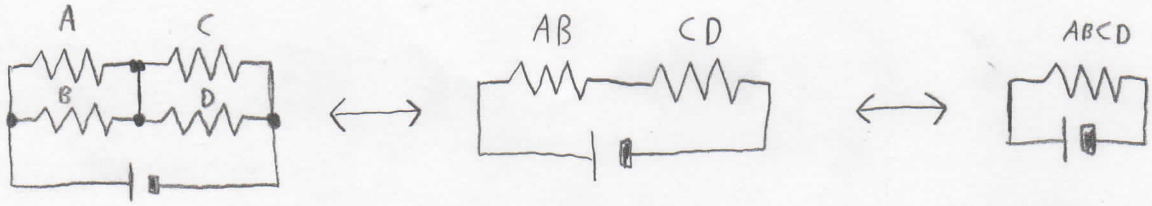
Just pick a direction. If you choose wrong, the current will have a negative value. Usually current will flow away from the positive terminal of the largest battery, but this is not always the case.

When applying the loop rule, how do you decide whether a battery has a positive or negative potential difference? How do you decide for a resistor?

For a battery, the potential difference will increase from negative to positive and decrease from positive to negative.

For a resistor, the potential difference will decrease in the direction of the current and increase against the direction of the current.

1)



a) $R_{eq} = R_{AB} + R_{CD} = \left(\frac{1}{R_A} + \frac{1}{R_B}\right)^{-1} + \left(\frac{1}{R_C} + \frac{1}{R_D}\right)^{-1} = 2.7\ \Omega$

b) Start from $P = \frac{V^2}{R}$ and use the simplified circuit diagrams to find the voltage.

$P_A = \frac{V_{AB}^2}{R_A} = \frac{(I_T R_{AB})^2}{R_A} = \left(\frac{V_T}{R_{eq}}\right)^2 \frac{\left(\frac{1}{R_A} + \frac{1}{R_B}\right)^{-2}}{R_A} = \left(\frac{3.9\ V}{2.7\ \Omega}\right)^2 \frac{\left(\frac{1}{2.0\ \Omega} + \frac{1}{7.0\ \Omega}\right)^{-2}}{2.0\ \Omega} = 2.5\ W$

$P_B = \frac{V_{AB}^2}{R_B}$ Since A and B are in parallel, the voltage is the same. $P_B = \frac{V_{AB}^2}{7.0\ \Omega} = 0.72\ W$

$P_C = \frac{V_{CD}^2}{R_C} = \frac{(I_T R_{CD})^2}{R_C} = \left(\frac{V_T}{R_{eq}}\right)^2 \frac{\left(\frac{1}{R_C} + \frac{1}{R_D}\right)^{-2}}{R_C} = 0.68\ W$

$P_D = \frac{V_{CD}^2}{R_D} = \frac{V_{CD}^2}{1.6\ \Omega} = 1.7\ W$

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2) a) Resistance is now $R_{eq} + r = 2.7\ \Omega + 0.1\ \Omega = 2.8\ \Omega$

So current through circuit is now $I_T = \frac{V_T}{R_{eq} + r} = \frac{3.9\ V}{2.8\ \Omega} = 1.4\ A$

Terminal voltage is $V_T = \varepsilon - I_T r = 3.9\ V - 1.4\ A \cdot 0.1\ \Omega = 3.76\ V$

b) Repeat the same process as problem 1(b), but use 3.76 V instead of 3.9 V.

$P_A = 2.3\ W \qquad P_B = 0.67\ W \qquad P_C = 0.63\ W \qquad P_D = 1.6\ W$

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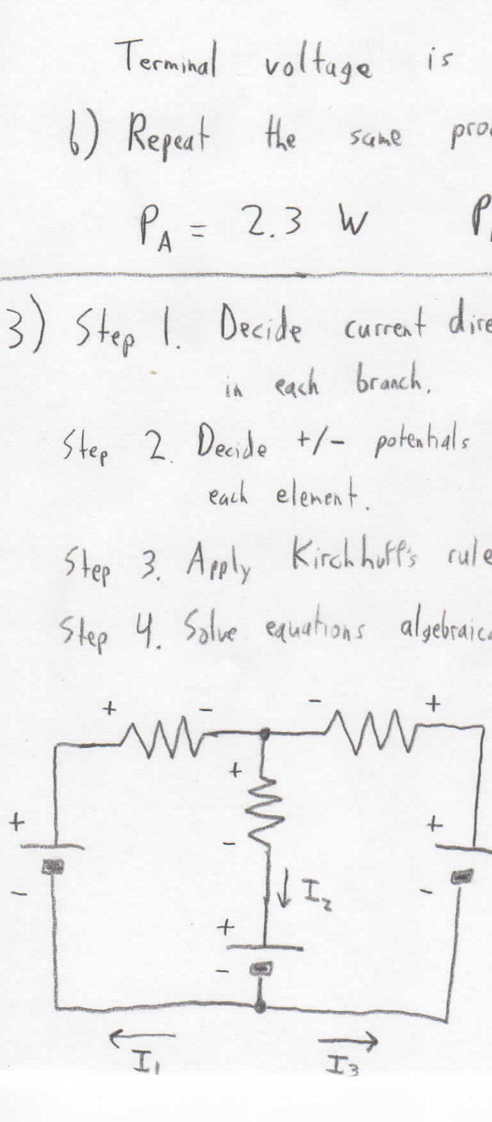
3) Step 1. Decide current direction in each branch.

Step 2. Decide +/− potentials for each element.

Step 3. Apply Kirchhoff's rules.

Step 4. Solve equations algebraically.



Junction rule: $I_1 + I_3 = I_2$ (1)

Loop rule: $7.0\ V - I_1 \cdot 0.8\ \Omega - I_2 \cdot 2.0\ \Omega - 3.0\ V = 0$ (2)

Loop rule: $7.0\ V - I_1 \cdot 0.8\ \Omega + I_3 \cdot 3.0\ \Omega - 5.0\ V = 0$ (3)

I chose loops  for (2) and  for (3). If you chose different loops or current directions, you will have different equations.

Subtract (2) from (3): $I_3 \cdot 3.0\ \Omega + I_2 \cdot 2.0\ \Omega - 2.0\ V = 0$ (4)

Substitute (1) into (4): $I_3 \cdot 3.0\ \Omega + (I_1 + I_3) \cdot 2.0\ \Omega - 2.0\ V = 0$ (5)

Solve (5) for I_1 : $I_1 = \frac{2.0\ V}{2.0\ \Omega} - \frac{2.0\ \Omega + 3.0\ \Omega}{2.0\ \Omega} I_3 = 1.0\ A - \frac{5}{2} I_3$

Solve (3) for I_3 : $I_3 = \frac{0.8\ \Omega}{3.0\ \Omega} I_1 - \frac{2.0\ V}{3.0\ \Omega} = \frac{4}{15}\left(1.0\ A - \frac{5}{2} I_3\right) - \frac{2}{3}\ A$

Solve (7) for I_3 : $I_3 = \left(\frac{4}{15} - \frac{2}{3}\right) A \cdot \frac{3}{5} = -\frac{6}{25}\ A$

I_3 is negative, so it actually points clockwise.

$I_1 = 1.0\ A - \frac{5}{2}\left(-\frac{6}{25}\right) A = \frac{8}{5}\ A$

$I_2 = I_1 + I_3 = \left(\frac{8}{5} - \frac{6}{25}\right) A = \frac{34}{25}\ A$