PH202-1G
Spring 2014

Worksheet February 5

Capacitors wired in parallel

When capacitors are wired in parallel, what do you know about the voltage across each capacitor and the charge stored by each capacitor?

The voltage across capacitors wired in parallel is the same (just like resistors in parallel).

The charge stored by capacitors in parallel is additive.

Give a mathematical expression for the equivalent capacitance of capacitors wired in parallel.

$$C_p = \sum_{i=1}^{n} C_i = C_1 + C_2 + C_3 + \ldots + C_n \quad \text{(like resistors in series).}$$

Capacitors wired in series

When capacitors are wired in series, what do you know about the voltage across each capacitor and the charge stored by each capacitor?

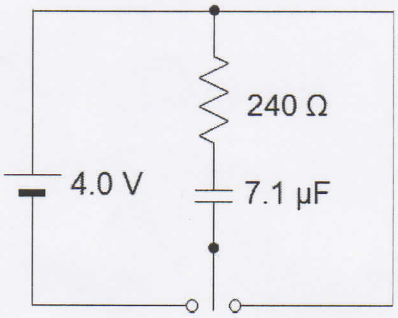
The charge stored by capacitors in series is the same (similar to current for resistors in series).

The voltage across capacitors in series is additive.

Give a mathematical expression for the equivalent capacitance of capacitors wired in series.

$$C_s = \left(\sum_{i=1}^{n} \frac{1}{C_i}\right)^{-1} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}\right)^{-1} \quad \text{(like resistors in parallel).}$$

RC circuits



If the capacitor in the circuit shown to the right contains no charge and the switch is closed to the left, what will happen?

The capacitor will begin charging until it contains an equilibrium charge $q_0 = CV_0$

If the capacitor contains a nonzero charge and the switch is opened, i.e. it is closed to neither the left nor the right, what will happen?

Nothing will happen. There is no complete circuit, so electrons cannot leave the charged plates of the capacitor.

If the capacitor contains a nonzero charge and the switch is closed to the right, what will happen?

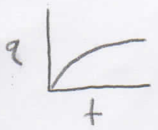
The capacitor will begin discharging until it contains no charge.

How is the time constant defined for an RC circuit? What does it represent for a charging capacitor and a discharging capacitor?

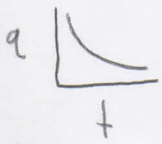
The time constant is defined as $\tau = RC$ for an RC circuit, with units of seconds.

It represents the time required for a charging capacitor gain 63% $(1-e^{-1})$ of its equilibrium charge and for a discharging capacitor to lose 63% of its initial charge.

Give a mathematical expression for the charge stored by a charging capacitor and a discharging capacitor in an RC circuit.



Charging $\quad q(t) = q_0\left[1 - e^{-t/\tau}\right] = CV_0\left[1 - e^{-t/(RC)}\right]$



Discharging $\quad q(t) = q_0 e^{-t/\tau}$

Be careful - q_0 does not necessarily equal CV_0 for a discharging capacitor.

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1. Three capacitors have capacitances of 120 nF, 80 nF, and 96 nF. If these capacitors are wired in series, what is their equivalent capacitance? If a potential difference of 3.0 V is applied across the capacitors, what is the charge stored by each capacitor and the voltage across each capacitor?

Capacitors in series add like resistors in parallel.

$$C_s = \left(\frac{1}{C_A} + \frac{1}{C_B} + \frac{1}{C_C}\right)^{-1} = \left(\frac{1}{120\ nF} + \frac{1}{80\ nF} + \frac{1}{96\ nF}\right)^{-1} = 32\ nF$$

The charge stored by capacitors in series is the same. $q = C_s V = 96\ nC$

$$V_A = \frac{q}{C_A} = \frac{96\ nC}{120\ nF} = 0.8\ V \qquad V_B = \frac{96\ nC}{80\ nF} = 1.2\ V \qquad V_C = \frac{96\ nC}{96\ nF} = 1.0\ V$$

2. The capacitors in problem 1 are now wired in parallel. Find their equivalent capacitance. If a potential difference of 3.0 V is applied across the capacitors, what is the charge stored by each capacitor and the voltage across each capacitor?

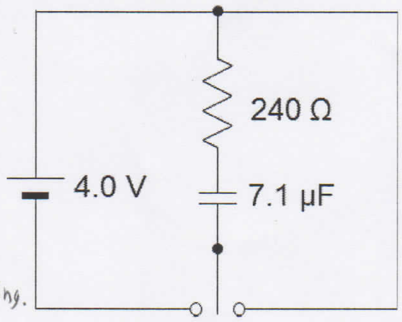
Capacitors in parallel add like resistors in series.

$$C_p = C_A + C_B + C_C = 120\ nF + 80\ nF + 96\ nF = 296\ nF$$

The voltage across each capacitor in parallel is the same. $V = 3.0\ V$

$$q_A = C_A V = 120\ nF \cdot 3.0\ V = 360\ nC \qquad q_B = 80\ nF \cdot 3.0\ V = 240\ nC \qquad q_C = 96\ nF \cdot 3.0\ V = 288\ nC$$

3. The capacitor in the circuit depicted to the right initially stores no charge. At time $t = 0$, the switch is closed to the left. After 3.0 ms, the switch is closed to the right. Find a) the amount of charge stored by the capacitor when the switch is closed to the right, and b) the time at which the discharging capacitor contains 10 μC of charge.



a) When the switch is closed to the left, the capacitor is charging.

$$q = q_0\left[1 - e^{-t/\tau}\right] = CV_0\left[1 - e^{-t/(RC)}\right]$$

$$q = 7.1\ \mu F \cdot 4.0\ V\left[1 - e^{-0.0030\ s/(240\ \Omega \cdot 7.1\ \mu F)}\right] = 24\ \mu C$$

b) Now the capacitor is discharging, so q from part (a) becomes q_0 in part (b).

$$q = q_0 e^{-t/\tau} \Rightarrow \ln\left(\frac{q}{q_0}\right) = -\frac{t}{\tau}$$

$$t = -RC\ln\left(\frac{q}{q_0}\right) = -240\ \Omega \cdot 7.1\ \mu F\ \ln\left(\frac{10\ \mu C}{23.52\ \mu C}\right) = 1.5\ ms$$

Don't round this number. ↗
Logarithms multiply error!