

Current-carrying wires in magnetic fields

What happens to a wire carrying a current when it is placed in a magnetic field?

The wire experiences a force in the direction determined by the right-hand rule.

Give a mathematical expression for the magnetic force on a current-carrying wire.

$$F_B = I L B \sin \theta$$

What happens to a loop of wire carrying a current when it is placed in a magnetic field?

The loop experiences a torque around an axis of rotation.

Give a mathematical expression for the torque on a coil of current-carrying wire in a magnetic field.

$$\tau = N I A B \sin \theta$$

What is magnetic moment? What are the units for the magnetic moment?

Magnetic moment $\mu = N I \underbrace{A}_{\text{area}}$ in ampere-square meters $A \cdot m^2$

Magnetic field produced by a current-carrying wire

What does the magnetic field around a current-carrying wire look like? How do you determine its direction?

The magnetic field around a wire is circular. If your right thumb points in the direction of the current, the magnetic field points in the direction of your fingers.

Give a mathematical expression for the magnetic field produced by a current-carrying wire.

$$B = \frac{\mu_0 I}{2\pi r}$$

What is the permeability of free space? What are its units?

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ N} \cdot \text{A}^{-2} = 1.256 \cdot 10^{-6} \text{ N} \cdot \text{A}^{-2} = 1.256 \cdot 10^{-6} \text{ T} \cdot \text{m} \cdot \text{A}^{-1}$$

How do two wires with currents pointing in the same direction interact? How do two wires with currents pointing in opposite directions interact?

Current in same direction - wires attract

Current in opposite direction - wires repulse

Give a mathematical expression for the magnetic force on a current-carrying wire due to the magnetic field produced by another current-carrying wire.

$$F_B = I_1 L_1 B_2 \sin \theta = I_1 L_1 \left(\frac{\mu_0 I_2}{2\pi r} \right) \sin \theta$$

Give a mathematical expression for the magnetic field at the center of a loop of current-carrying wire.

$$B = \frac{N \mu_0 I}{2R}$$

Give a mathematical expression for the magnetic field within a solenoid. Under what condition is this expression valid?

$$B = \frac{\mu_0 N I}{L} \quad \text{This expression is valid if } L \gg R$$

Problems

1. A loop of wire with a length of 2.0 cm and a width of 2.0 cm is attached to a rod such that it can rotate freely about the axis of the rod. The loop is placed in a magnetic field of strength 0.4 T oriented perpendicularly to the axis, and a current of 2.5 A is passed through the loop. A second loop with a length of 2.0 cm and a width of 3.0 cm is placed in the same magnetic field, and a current of 1.5 A is passed through the second loop. a) Which loop experiences a larger torque? b) How do the final orientations of the two loops differ?

$$\begin{aligned} a) \quad \tau_1 &= I_1 A_1 B \sin 90^\circ = 2.5 \text{ A} (2.0 \text{ cm} \cdot 2.0 \text{ cm}) \cdot 0.4 \text{ T} = 4.0 \cdot 10^{-4} \text{ N}\cdot\text{m} \\ \tau_2 &= I_2 A_2 B \sin 90^\circ = 1.5 \text{ A} (2.0 \text{ cm} \cdot 3.0 \text{ cm}) \cdot 0.4 \text{ T} = 3.6 \cdot 10^{-4} \text{ N}\cdot\text{m} \end{aligned}$$

The first loop experiences a greater torque.

b) The final orientations will be the same - perpendicular to the magnetic field.

2. A 80-cm long wire carries a current of 0.16 A. A second wire with the same length carrying a current of 0.32 A is placed 30 cm away from the first wire, oriented parallel to the first wire. a) If the wires experience a repulsive force, does the current in the second wire point in the same direction as or a different direction from the current in the first wire? b) Calculate the magnetic force on each wire.

a) Repulsive force indicates that the current points in the opposite direction.

b) Magnitude of the force on the two wires is the same.

$$F_B = \frac{\mu_0 I_1 I_2 L}{2\pi r} = \frac{1.256 \cdot 10^{-6} \text{ T}\cdot\text{m}\cdot\text{A}^{-1} (0.16 \text{ A}) (0.32 \text{ A}) (0.80 \text{ m})}{2\pi \cdot 0.30 \text{ m}} = 2.7 \cdot 10^{-8} \text{ N}$$

3. A current of 24.0 A is passed through a coil of wire with radius 15 cm. At the center of the wire, the magnetic field is measured to be $6.03 \cdot 10^{-3}$ T. How many turns are in the coil?

$$N = \frac{2R B}{\mu_0 I} = \frac{2(0.15 \text{ m})(6.03 \cdot 10^{-3} \text{ T})}{1.256 \cdot 10^{-6} \text{ T}\cdot\text{m}\cdot\text{A}^{-1} (24.0 \text{ A})} = 60$$