PH202-1G
Spring 2014

Worksheet February 24

Inductance

What happens when a current is run through a coil of wire which lies close to a second coil of wire?

The First (primary) coil induces an emf in the secondary coil (mutual induction) and induces an emf in itself (self-induction)

Give a mathematical expression for the emf induced in the secondary coil of wire by mutual induction.

$$emf_s = -M \frac{\Delta I_p}{\Delta t}$$ where ΔI_p is the change in current in the primary coil.

Define mutual inductance and give its units of measurements in terms of other units you have studied.

$$M = \frac{N_s \Phi_s}{I_p}$$ in henries (H), where $1\ H = 1\ V \cdot s \cdot A^{-1}$

Give a mathematical expression for the emf induced in the primary coil of wire by self-induction.

$$emf_p = -L \frac{\Delta I}{\Delta t}$$ where ΔI is the change in current in the primary coil.

Define inductance (sometimes called self-inductance) and give its units of measurement.

$$L = \frac{N \Phi}{I}$$ in henries (H)

Give a mathematical expression for the energy stored in an inductor.

$$\text{Energy} = \frac{1}{2} L I^2$$

Transformer

Describe qualitatively what a transformer does? How does it affect current, voltage, and average power?

Transformers increase or decrease the voltage in a coil of wire using mutual induction. They cause an opposite change in the current and do not change average power.

Give a mathematical expression for the activitiy of a transformer.

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{I_s}{I_p}$$

RLC circuit

Define the inductive reactance and the capacitive reactance and give their units of measurement.

Inductive reactance $X_L = \omega L$ and capacitive reactance $X_C = \frac{1}{\omega C}$, both measured in ohms.

Give a mathematical expression for the root-mean-square voltage in a capacitive and inductive ac circuit.

Inductive circuit: $V_{rms} = I_{rms} X_L$ Capacitive circuit $V_{rms} = I_{rms} X_C$

Describe the relationship between the phase of the current and the phase of the ~~velocity~~ voltage in a resistive, capacitive, and inductive ac circuit.

Resistive circuit – current is in phase with voltage
Capacitive circuit – current leads voltage by $\frac{\pi}{2}$
Inductive circuit – current lags voltage by $\frac{\pi}{2}$

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Define the impedance of an RLC circuit and give its units of measurement.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Give a mathematical expression for the root-mean-square current in an RLC circuit. At what frequency is this current at a maximum?

$$I_{rms} = \frac{V_{rms}}{Z} \qquad \text{Resonant Frequency} \quad f_R = \frac{1}{2\pi\sqrt{LC}}$$

Problems

1. The ac voltage received from a standard US outlet is 120 V. The evil Dr. Horrible needs an ac voltage of 500 V to power his death ray and bring havoc to the city. By what factor should Dr. Horrible increase the number of turns in secondary coil compared to the number of turns in the primary coil in a transformer in order to achieve the required voltage? If the current produced by the wall socket is 15 A, how much current flows through the death ray?

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{500\text{ V}}{120\text{ V}} = 4.17$$

Since voltage is stepped up, current will be stepped down.

$$I_s = \frac{V_p}{V_s} I_p = \frac{120\text{ V}}{500\text{ V}} \cdot 15\text{ A} = 3.6\text{ A}$$

2. An RLC circuit consists of an ac voltage source with a peak voltage of 12 V, a resistor with a resistance of 8.0 Ω, a capacitor with a capacitance of 150 μF, and an inductor with an unknown inductance. The root-mean-square current through the circuit reaches its maximum value at a frequency of 600 Hz. What is the magnitude of the ~~maximum value of the~~ root-mean-square current, at half of this frequency?

First, we need to find the inductance using $f_R = \frac{1}{2\pi\sqrt{LC}}$ for the resonant frequency.

$$L = \frac{1}{(2\pi f_R)^2 C} = \frac{1}{(2\pi \cdot 600\text{ Hz})^2 \cdot 150\ \mu\text{F}} = 4.69 \cdot 10^{-4}\ \text{V} \cdot \text{s} \cdot \text{A}^{-1} = 469\ \mu\text{H}$$

Then, find impedance using X_L and X_C at the desired frequency.

$$X_L = \omega L = 2\pi \cdot 300\text{ Hz} \cdot 469\ \mu\text{H} = 0.884\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 300\text{ Hz} \cdot 150\ \mu\text{F}} = 3.54\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(8.0\ \Omega)^2 + (0.884\ \Omega - 3.54\ \Omega)^2} = 8.43\ \Omega$$

Now we can find I_{rms} from Z and $V_{rms} = \frac{V_0}{\sqrt{2}}$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_0}{\sqrt{2}\,Z} = \frac{12\text{ V}}{\sqrt{2} \cdot 8.43\ \Omega} = 1.0\text{ A}$$