PH202-1G
Spring 2014

Worksheet January 29

Alternating current

What is the difference between direct current and alternating current?

Direct current flows in one direction.

Alternating current oscillates between a positive and negative peak value.

Give a mathematical expression for the voltage provided by an ac power source.

$$V = V_0 \sin(2\pi f t)$$

Give a mathematical expression for the current through an ac circuit.

$$I = I_0 \sin(2\pi f t)$$

For both of these expressions, the argument for the sine function must be <u>radians</u>, not degrees

What is an rms value? Why do we use rms values when talking about ac circuits?

$$X_{rms} = \frac{X}{\sqrt{2}}$$

X can be anything — current, voltage, etc.

Rms values make the math easier when calculating averages, which are useful for an ac circuit which is always changing.

Give a mathematical expression for the average power consumed by an ac circuit.

$$\overline{P} = \overline{IV} = \overline{I_0 V_0 \sin^2(2\pi f t)} = \frac{1}{2} I_0 V_0 = \frac{I_0}{\sqrt{2}} \frac{V_0}{\sqrt{2}} = I_{rms} V_{rms}$$

The average value of $\sin^2\theta = \frac{1}{2}$

Resistors wired in series

When resistors are wired in series, what do you know about the current through each resistor and the potential difference across each resistor?

For resistors wired in series, the current through each resistor is the same and $V_i = I R_i$

Give a mathematical expression for the equivalent resistance of resistors wired in series.

$$R_s = \sum_{i=1}^{n} R_i = R_1 + R_2 + R_3 + \ldots + R_n$$

R_s will always be bigger than any single resistor.

Resistors wired in parallel

When resistors are wired in parallel, what do you know about the current through each resistor and the potential difference across each resistor?

For resistors wired in parallel, the potential difference across each resistor is the same and $I_i = \frac{V}{R_i}$

Give a mathematical expression for the equivalent resistance of resistors wired in parallel.

$$R_p = \left(\sum_{i=1}^{n} \frac{1}{R_i}\right)^{-1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}\right)^{-1}$$

R_p will always be smaller than any single resistor.

For two resistors wired in parallel:

$$R_p = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2}\right)^{-1} = \left(\frac{R_1 + R_2}{R_1 R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

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Problems

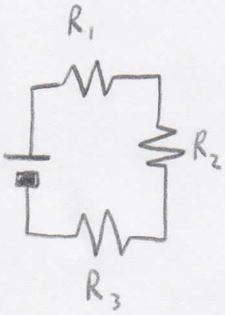
1. A circuit is driven by an ac power source with peak voltage of 1.5 V. At 118 ms, the voltage provided by the power source is 0.65 V. With what frequency does the power source oscillate? If the average power is 3.2 W, what is the peak current in the circuit?

$$f = \frac{\arcsin\left(\frac{V}{V_0}\right)}{2\pi t} = \frac{\arcsin\left(\frac{0.65\,V}{1.5\,V}\right)}{2\pi \cdot 0.118\,s} = 60\ Hz$$

Make sure you use radians!

$$I_0 = \frac{2\bar{P}}{V_0} = 4.3\ A$$

2. Three resistors have resistances of 4.1 Ω, 1.8 Ω, and 2.3 Ω. If the resistors are wired in series, what is their equivalent resistance? If a potential difference of 5 V is applied to the resistors, what is the current and voltage across each resistor?



$$R_s = R_1 + R_2 + R_3 = 4.1\,\Omega + 1.8\,\Omega + 2.3\,\Omega = 8.2\,\Omega$$

Current through each resistor is same

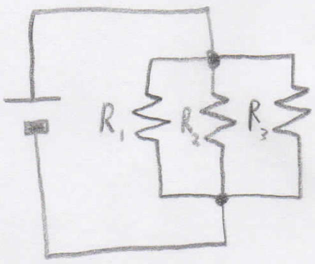
$$I = \frac{V}{R_s} = \frac{5\,V}{8.2\,\Omega} = 0.61\ A$$

$$V_1 = IR_1 = (0.61\,A)(4.1\,\Omega) = 2.5\ V \qquad V_2 = IR_2 = 1.1\ V$$

$$V_3 = IR_3 = 1.4\ V \qquad \text{Check: } V_1 + V_2 + V_3 = 5\ V$$

3. The resistors in problem 2 are now wired in parallel. Find their equivalent resistance. If a potential difference of 5 V is applied to the resistors, what is the current and voltage across each resistor?

$$R_p = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{4.1\,\Omega} + \frac{1}{1.8\,\Omega} + \frac{1}{2.3\,\Omega}\right)^{-1} = 0.81\,\Omega$$



Voltage across each resistor is same. $V = 5\ V$

$$I_1 = \frac{V}{R_1} = \frac{5\,V}{4.1\,\Omega} = 1.2\ A$$

$$I_2 = \frac{5\,V}{1.8\,\Omega} = 2.8\ A \qquad I_3 = 2.2\ A$$

$$\text{Check: } I_T = \frac{V}{R_p} = \frac{5\,V}{0.81\,\Omega} = 6.2\ A = I_1 + I_2 + I_3$$